An observation on Supersymmetry and the Wilczek-Zee model

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Abstract

Motivated by the almost exact match of composite supersymmetry, we look for hidden supergenerators in the Wilczek-Zee model.

The experimental situation during the development of GUT and SUSY theories was different from the actual one. At these times the neutrinos were massless, and the top was expected to be less than 40 GeV. The use of SO(10), with a massive neutrino, was thought to be a prediction, and SU(5) the safe bet.

Massive neutrinos change the situation because 32 degrees of freedom for each generation are easier to fit than 30. Another not usually noticed point is that a massive top quark with exactly three generations leaves only five flavours to build bound states of SU(3). This is already interesting in the world of SUSY QCD, where $N_f = N_c + 2$ is known to be a transition between two regimes.

But is is even more noticeable as we consider Standard Model charges. The 96 degrees of freedom of the spinors of the Standard Model divide in 24 from the leptons plus 72 from the quarks. Five flavours of quarks can be combined with antiquarks to form uncoloured $(q\bar{q})$ pairs in 25 different ways. If we look at the charges of u, c and d, s, b, we see that 6 of these pairs have charge $+1$, another 6 have charge -1 and 13 are neutral. So they are very near of coinciding with the 6, 6, 12 degrees of freedom of the respective leptons.

We turn our attention to the quark sector, and we see that $N_f = 5, N_c = 3$ can combine to give a total of 45 pairs of (qq) , and of course another 45 of $(\bar{q}\bar{q})$. So we are a bit over the coloured d.o.f in this case. On closer attention, looking at the electric charge we see that of the 45 pairs, 18 are of charge $-2/3$, 18 are of charge $+1/3$, and 9 have the more exotic charge $+4/3$. Had some way to discard the charges $\pm 4/3$, we would have exactly 72 possible combinations for coloured pairs of particles, and the right number for each charge.

To resume: for electrically charged leptons and quarks, the number of degrees of freedom matches the number of possible combinations of SU(3)-binded pairs.

Thus we can suspect that the motto *"No supersymmetric partners are observed"*, coined in the late seventies, is not true now that we know that there is only 3 generations, that the top quark is very massive and that neutrinos have doubled their degrees of freedom.

We can suspect there is an slightly broken N=1 supersymmetry between the fermions of the standard model and their SU(3)-binded bosons.

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Of course there are some obstructions to address. On a straightforward diquark construction, the (uc) scalar is allowed, and we do not want it (neither the (uu), (cc) diquarks). And probably the required (dd),(ss),(bb) spin 0 bosons would not be rightly symmetrised. So we need some extra mechanism independent of the family number and asking for different symmetrisation conditions in the up combinations $(uu)(cc)(uc)$ than in the down (dd) (ss) (bb) (ds) (db) (sb) combinations, so that the former become forbidden and the later become safely allowed. Considering that in usual phenomenological susy models, as the MSSM, the up and down sectors get mass from different particles, it does not seem very unlikely the existence of such mechanism. Another obstruction, experimental, is the non-observation of the charge $-2/3$ diquarks (dd) and (ss). If they can not really be extracted from the measured particle spectrum, we can always assume their mass is partner of the mass of the (anti)top.

Having supersymmetry to composites helps to explain some features of the charged leptons of the standard model. The pion mass is near of the muon mass. And all the three leptons fulfill a mass formula, Koide's formula[2], that was designed for composite objects.

What kind of supersymmetry do we have? Given the above match, it is clear that bosons will be kept as scalars or pseudoscalars; beyond spin 0 they would produce an excesive number of degrees of freedom. And in the fermion side, we are pretty sure we only have these spin 1/2 elementary objects. So $N = 1$ supersymmetry seems in principle our only option. Still, it is intriguing to notice that a full generation multiplet would contain 64 degrees of freedom (32 fermionic and 32 bosonic). We will devote the rest of this letter to discuss the possibility of arranging the known SM spectrum to imitate $N > 1$ supermultiplets.

Had we 256 total degrees of freedom, we should look for a $N = 8$ supergravity multiplet. Regrettably, with three generations we are in middle ground, 128+64, and we do not know really if we are interested on massive or massless states. The former could be useful if we do not rely in a SM Higgs mechanism, the later are simpler to work out. So, in the following discussion, consider "$N = \ldots$" as a helpful label for an alternating sequence of fermionic and bosonic states. To fix the argument, lets proceed with massless irreps, no central charges.

Lets think first how should we arrange a single generation. It could be a $N = 6$ object with fermionic and bosonic degrees of freedom alternating in a multiplet of size

$$1\ 6\ 15\ 20\ 15\ 6\ 1$$

Textbooks like to say *"we are going to study N=1 because it is the more general case"*. Indeed we could build this supermultiplet by joining smaller supermultiplets of the kind $(s, s+1/2)$. Obviously we need to use 1 pair between 1 and 6, 5 pairs between 6 and 15 and so on. The sequence of fermions would be

$$1, 5, 10, 10, 5, 1$$

At this point, the temptation of using the **5** and **10** representations of SU(5) is powerful and there are no reasons to resist. So our list could start:

1	ν		
6	$(q\bar{q})$	$(q\bar{q})(u\bar{d})\,(\bar{d}\bar{u})_{\{r,g,b\}}$	
15		$\nu\, e^+\, d_{\{r,g,b\}}$	$d_*\, u_*\, \bar{u}_*\, e^+$
20	.		...
15	.		
	.		

Of the fermions of the third line, five will pair with the bosons in the second line, and ten will pair with the bosons in the fourth. Note that there are some variations about which **5** and which **10** to put first, but no more than four different possibilities at all. The one we have chosen implies that besides the charge-preserving $N = 1$ transformation of the neutrino to some $(q\bar{q})$ we should look for new supersymmetry generators transforming the neutrino to another extra neutral meson, to a charged $(u\bar{d})$ q=+1 uncoloured meson and to three $(\bar{d}\bar{u})$ q=-1/3 colored diquarks. Thus these new susy generators can carry both electric and colour charge. In fact we see that they carry the needed charges to produce **5** of SU(5) and, when iterated, to produce the **10**. I have not found in the literature the explicit suggestion of using supergenerators with $Q^2 = 0$ to produce the representations of SU(5), but it seems a natural process.

The charges of the supergenerators, and here the main point, are the same than in the composite version of Wilczek-Zee "binary" model [1], and in fact a "+" in the bit array of the WZ model is equivalent to the use of the corresponding supergenerator; we have one extra neutral generator from our original susy, but is a price we can gladly to pay if in this way we can speak of supergenerators instead of preons.

The WZ model, as the authors themselves explain, is a way to exploit the SU(2) "spin" decomposition of $SO(2k)$ in a notation that could be related to compositeness. We have refocused this "spin" in order to exploit supersymmetry. It is amusing that at the same time we seem to need compositeness too, but we can survive with the Standard Model fermions.

What does it happen with the physical spin? Well, certainly we need fermions going to bosons and back, but I am not sure about the need of higher spin. If we produce N=4 from extra dimensional N=1, the charged generators come from the extra dimensions, not surprisingly if you think in Kaluza-Klein terms. Should rotations that are only visible in the extra dimensions have some effect in angular momentum of the four dimensional theory?

Now for three generations. Here we are lost, but let me to sketch where we stand. Remember that we really need to consider three generations, because it is the only way to get the right number of bosonic degrees of freedom.

We could just consider three similar $N = 6$ multiplets. But we can also start from different points in order to almost mimic a $N = 8$ object

	1	6	15	20	15	6	1	
1	6	15	20	15	6	1		
		1	6	15	20	15	6	1
1	7	22	41	50	41	22	7	1

In some way, it is the substraction of $N = 8$ minus $N = 6$. Of course we still want fermions on one side, bosons in another. What means, that in doing the sum we must exchange the fermions and bosons of the N=1 susy in order to be sure we are not adding pears with apples.

The same argument than in the one generation case invites us to look for representations from the sum

0	1	5	10	10	5	1	0
1	5	10	10	5	1	0	0
0	0	1	5	10	10	5	1
1	6	16	25	25	16	6	1

but I have never seen a **25** of a simple group.

So no hint from group theory this time. If, on the other hand, we try to add supergenerators straightforwardly, we are stuck too: one generator allow us to change only from one family to a second one, and two generators would produce not three but four families. Not to count that in most cases an odd number of generators automatically upgrades itself to a bigger multiplet.

We could look for help in the new Kovtun-Zee model [3], evolved from W-Z to try to fit the three generations. But we could also remain happy that two generations can be mixed and another one stands apart, because it seems very much that the third generation must support different properties than the other two.

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