Standard Model Masses and Models of Nuclei

Alejandro Rivero

EUPT (Universidad de Zaragoza), 44003 Teruel, Spain

Abstract

In nuclear levels, the subshells responsible for doubly magic numbers happen to start their filling with nuclei having the same mass that relevant Standard Model bosons. Thus, via an undetermined many body effect, these bosons could actually contribute to the nuclear force.

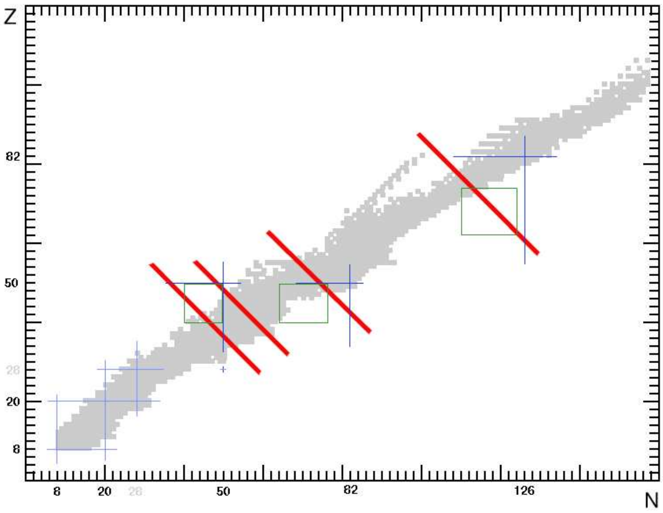

FIG. 1: Crosses mark doubly magic numbers. Diagonal isobars correspond to the masses of W,Z, H [3] and Top. Rectangles mark zones of orbital filling, with filling order from [8], for the spin-orbit depressed subshells.

INTRODUCTION

The measurement of the highest masses in the standard model, namely those of the top quark and the particles W and Z, shows an intriguing coincidence: the mass of the top occurs slighly before than the mass of the doubly magic nucleus of 208 Pb, while the mass of Z and W occur slighly before of the mass of 100 Sn.

These two magic numbers need of an anormally high spin-orbit splitting, and there is only a third doubly magic number sharing this need: the one of 132 Sn. Then, the ALEPH preliminary measurement [3] of a mass of 115 GeV for the Higgs boson, just slighly before the mass of this Tin isotope, makes imperative to report this triple coincidence and to state the problem, which I do in this letter. Due to the continous action of strong nuclear forces, the weak bonding aproximation is not valid in this context, so it is pausible that we are seeing some kind of general mesoscopic effect coming from many-body quantum mechanics, and not just a specific effect of nuclear physics.

It is known that magic numbers in nuclei are generated by separating levels via spin-orbit coupling. Low numbers, as the 28 or 20 in doubly magic Ca48 or Ni56, can be fit using traditional potentials with a relatively weak spin-orbit splitting, if any. Numbers 50, 82 and 126 are got because the respective subshells 5g9/2, 6h11/2 and 7i13/2 get large gaps and fall into the lower shell. Traditionally this was obtained via a purely phenomenological Hamiltonian, consisting of an empirical spin-orbit coupling and a Nilsson term $+k_n j^2$ depending of the shell. Modernly a relativistic approach based in meson exchange lets one to derive the spin orbit term from first principles. Still, the models are restricted to low nucleon numbers, because of computational effort. Some high (N,Z) models have been essayed, but they fail to reproduce the phenomena of double magicity and its associated quenching.

Here we see that some empirical coincidences are related to the availability of virtual J=0 and J=1 particles from the Standard Model. They seem to imply a need to correct meson exchange by adding the whole set of standard model interacting particles. Such particles should be noticeable during the process of filling the subshells responsable of the magicity, thus they occur slightly before of the double magic numbers.

Let me to organise the letter in small sections, so I will first comment on global issues, then to look into each level in detail, and finally to do some remarks about their possible significance.

MASS LANDSCAPE

There are four highly massive particles in the standard model: the vector bosons W,Z, the scalar Higgs boson, and the Top quark. This one can not appear free but in composites, the simplest being the bosons $(t\bar{u})$, $(t\bar{d})$, etc, all of them having a mass near to the Top mass. Thus we will refer to all these particles as "the standard model bosons".

If we draw the traditional plot of energy per nucleon, for instance for the Audi-Wapstra experimental tables, we notice three peaks[14] in the descending part. Converting from GeV to atomic mass units, we can draw over the plot four lines indicating the respective masses of W, Z,Higgs (conjectured by LEP-2), and Top. Then we note that boson masses happen slightly before the peaks.

Now, the peaks in such plot are mostly due to the neutron magic numbers, but if one remembers that inside each neutron line 50, 82, 126 there is a doubly magic number, then it is more sensible to examine the proximity of the SM masses to the doubly magic numbers. This is done in figure 1, by using the traditional N-P plot of nuclides. Here we plot diagonal isobaric lines for each massive particle, and we see that effectively the lines happen near the crosses corresponding to doubly magic numbers.

In order to determine how near is "near", we need to add some information to the plot. Namely, we will consider the subshell structure before each double magic number.

Also, as our masses are incorporated directly from the standard model, without direct relation to a nuclear model, we could expect to find some signal of them in mass systematics. Indeed the droplet model FDRM 1992 [10], which is the state-of-the-art mic-mac model, shows strongly this signal for the W,Z, and a weaker error signal for higgs and top, where a quadrupole correction dominates the adjust.

a)P=82, N=126

The mass of the top quark was measured by the Tevatron in 1994, and it amounts, by direct observation, to 174.3 GeV, about 187.1 amu.

The proton magic number 82 is due to the 6h11/2 subshell, which actually is under two other levels, the 3s1/2 and the 4d3/2. The neutron magic number 126 is due to subshell 7i13/2. If we use Klinkenberg 1952 filling scheme [8] to draw the rectangle of nuclei with partial fillings of (6h11/2,7i13/2), the diagonal approaches closely to the t quark isobaric line. The situation is shown in figure 2.

We should alert the reader that the filling scheme is not well defined because the energy difference between odd and even number of nucleons is enough to alter the energy levels. Thus if we get the shell scheme for 208Pb from Bohr-Mottelson 1969[15], then the levels 5f5/2 and 4p3/2 seem to raise above the 7i13/2, and then the isobar line just cuts a corner of the rectangle. This kind of complications are usual and we will find the same issue in the N=82 range.

One should consider also that a 5 GeV range is available starting from the top mass, if we want to consider here all the "mesons" from $t\bar{d}$ to $t\bar{b}$. Of course, the highly unstable $t\bar{t}$ "meson" has a mass far away from the nuclear data.

b)P=50, N=50

Here we still have the same subshell for protons and neutrons, namely the 5g9/2. Thus the relevant nuclei are in a square.

The masses of W and Z were measured by CERN collaborations in 1982-3. They amount to 80.423 and 91.1876 GeV, resp. 86.3 nd 97.9 amu.

While both masses are inside the square bracketed by the subshells, some models could also be interested in its average, 92.1 amu, which roughly closes the diagonal of the square.

If one draws the mass lines over the plot of errors in the microscopic-macroscopic mass

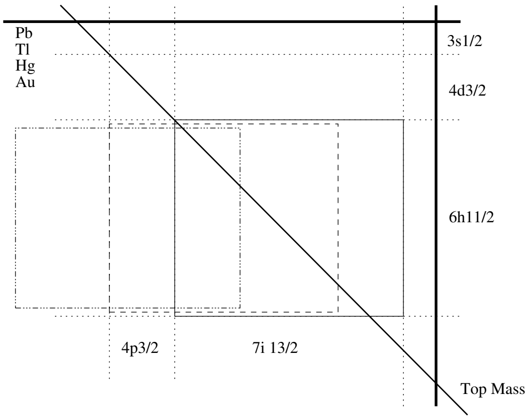

FIG. 2: Diagonal isobar signals the mass of the top quark. Levels and solid rectangle according scheme from [8]. Other possibilities are drawn as dashed rectangles

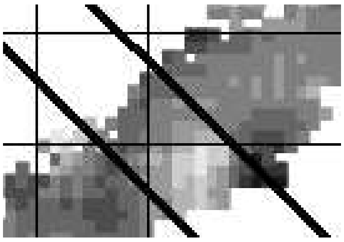

FIG. 3: Horizonal (vertical) lines mark start and end of the 5g9/2 subshell for protons (neutrons). Thus upper-right corner is doubly magic 50,50. Diagonal isobars are drawn at masses of W and Z. The background shows FDRM-1992 mass error [10] in this area of the nuclide table being black +1.5 MeV, white -2.0 MeV

formula FRDM-1992[10], it can be noticed that outside from the square the lines coincide with an huge error area in the model. In figure 3 we have included this data.

c)P=50, N=82

Even without the higgs, the two previous coincidences should be enough to consider an extension of nuclear models. Now, a Higgs-like event was reported by LEP-2 collaborations in their final year 2000 run, with a mass of 115 GeV; roughly 123.46 amu. We can plot it and see how near it is of the extant doubly magic number.

The magic number P=50 is due to the subshell 5g9/2 as in the previous case. The magic number N=82 is due to subshell 6h11/2. This should do a 10 × 12 rectangle.

An unexpected complication is the overlapping of neutron subshells 6h11/2, 3s1/2 and 4d3/2. Single nucleons prefer one subshell, while paired nucleons prefer another. It is

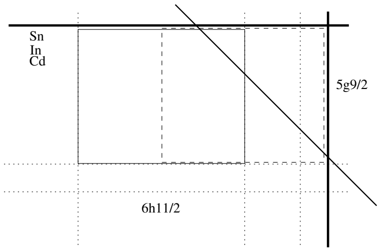

FIG. 4: Diagonal isobar marks the conjectured mass of the SM Higgs scalar. solid rectangle takes subshell filling order from [8], dashed rectangle takes subshell ordering from HFB models.

difficult to establish the right order, if it exists. Even when the 6h11/2 subshell is supposed to be under the other two (or three!), the first excited level is always provided by it.

If we ignore the overlap and we we accept the old criteria from Klinkenberg, putting the 6h subshell as the lower one, then the LEP-2 event simply cuts a corner of the rectangle. The same discrepancy, at a smaller scale, happens if we take Bohr-Mattelson. And if we draw a whole elongated rectangle accounting for all the possible occupations, then it is not so clear where should we draw the diagonal. In any case, we have reflected all these possibilities in figure 4.

On other hand, we should point out that all the modern models based on HFB mass formulae fail to reproduce the narrow overlap and the priority of 4d3/2, and they assign the highest energy level to the 6h11/2 subshell, which then closes the whole shell (check the plots and data in [4]). This should be the best approach, as then the 10×12 rectangle is directly attached to the doubly magic corner.

The area near this mass isobar has been studied also beyond Z=50, in the context of Chiral Doublet Structures [13]

ANALYSIS AND REMARKS

Our empirical approach is partly mudded because of the proximity between different subshell splittings. We refer the reader again to [8] to get an idea of the complexity of these determinations. On the other hand, it could be that all the levels are actually trying to compete to fit in the isobaric lines.

While it is possible to believe that this effect is just a random numerological coincidence, it covers all the cases where the naive shells plus spin-orbit do not suffice to justify the observed magicity, and specially it suggests that a motivation can exist for the effect of double magicity. There are not more highly massive Standard Model particles, and there are no more double magic nuclei with high spin-orbit splitting. It could be argued that the 28 shell is not included in the list, but this shell needs only a mild spin-orbit split, at the reach of classical models. It is very unusual to find such level of agreement in numerology with congruent units: most numeric coincidences appear in adimensional considerations, or when comparing numbers coming from different systems of units..

The strong nuclear force, as driven by the usual system of meson exchanges, provides a kinematical glue becouse our forces are small compared to the coupling constant from pions. We are working with bound states; any momenta exchanged between a bound particle and the nucleus is shared between all the particles of the nucleus before the "orbiting" particle gets able to turn around and interact from the opposite direction. So the acceleration from this momentum exchange depends on the mass of the whole nucleus, and this whole mass is more relevant that the mass of the nucleon -or quark- actually responsible of the exchange. It happens already in some classical inelastic collisions, where the conversion of energies $collision \rightarrow internal + mass\ center$ depends on the coupling constant of the internal force. The perturbation is to be measurable because in a yukawian potential it depends on the product of mass (inverse range) times coupling.

In any case, if the recoil depends of the whole mass, it is difficult to justify why should one to cut-off the calculation at the scale of the single nucleon. Regretly, most meson theories are effective QFT with a cutoff $\Lambda < 1$ GeV, so they are not directly suitable to incorporate the highest particles.

The enhancement happens in the depressed subshells, which agrees with the phenomenological approach that enhances the spin-orbit separation by incorporating a dependence on angular momenta. Still, We can not, at this moment, provide an explanation of the effect. The simultaneous fit of both proton and neutron subshells suggest that the effect includes an isospin symmetry.

For the top quark mesons, both J=1 and J=0 particles are available. More specifically[16] the pseudoscalar 0-+ should be expected with more probability that the scalar 0++, while the vector 1-- is always "the backbone of any meson spectroscopy". For the W and Z, we should look for axial effects influencing angular momenta, and also for other combinations explaining the role of W and Z in the error data of macroscopic models. If the LEP-2 event is confirmed to be the Higgs scalar particle, then we should look for couplings to J=0 scalar. It should be remembered that $J = 0$ coupling is the trademark of the scalar σ-meson in relativistic models of the nucleus. The status of this σ meson is experimentally troubled. It is usually assimilated to a $f_0(600)$ boson (see [11, pg 450]) by the HEP community, but nuclear authors prefer to remark that "there is no experimental evidence for a free σ meson, although the σ field is a crucial ingredient of relativistic mean-field models" [1]. Usually the σ scalar is seen as a two-pion particle, ie its scalar nature comes from two identical pseudoscalar couplings.

Our particles directly from the Standard Model could interfere with the effect of the σ. It could happen either as an small modification of the $1/m_\sigma$ mass term, or as a direct interaction with the nucleon density. It seems from our data that the meson exchange interaction is enhanced when the mass of the whole ensemble $N + Z$ of nucleons equals the mass of some available SM boson. This could be more explicit in the second case.

Also, it could be possible to develop some influence in the effective nucleon mass, which, at the end, is the main responsible for the spin-orbit coupling.

On other hand we could be seeing a quantum many-body effect. The interaction propagated via a massive particle will depend both on the mass of the nucleon, M, and the mass m of the boson. But there is also a multiplicative factor depending of the number of nucleons, i.e. of M_n/M, being M_n the mass of the nucleus. So the total interaction can be writen in terms of (M_n, M, m) and we could expect some surprise[17] when $M_n \approx m$. Besides, M_n could be directly the relevant quantity for kinematics, as explained above.

With a clear background, and in the absence of serious Effective Field Theory motivations, the ratio Γ/m between pole width and pole mass of a given meson should be a consideration more fundamental than simply to look if such mass is above or below the one of the proton. It should be of some help if similar effects were searched in few nucleon systems. The scale of J/psi, B and Upsilon could be an extra contribution respectively related to the variations of mass at 4He, 8B, 12C, or to contribute to the valleys between, or simply to be hidden under other effects. Furthermore, perhaps here the nucleon number is too small or the background of other particles too big. So the most we can say is that the low mass range does not contradict our observation, but that it is not a definitive support. Ab-initio models could be able to fit the data, in the future, without requiring the midly massive mesons.

Beyond theoretical frameworks, another clue to this effect can arrive from an analysis of the errors in current models of masses. Anomalies are to be expected near mass numbers 86, 98, 123, 187 (W, Z, Higgs, tops). We have remarked this point in figure 3. Indeed the error analysis of [5] for the same model hints the same results, and also the variations between theoretical and real error in fig 7 of [10]. Besides, the recent analysis of Huertas for FST model shows slope changes in the error at this mass numbers, see figure 4 of [6] and fig. 13 of [7].

Of course it shall not escape to any reader that a Mic-Mac model including high SM masses as free parameters could be used to predict these SM values, via least squares or any other variational fit. For instance one can calculate the model parameters dependence on the masses of the nuclei used in the fit; say $g_>(m)$ fitting to all nuclei greater than m, or $g_<(m)$ fitting for all the nuclei with mass smaller than m, or even some intermediate fit $g_{+10}(m)$. Then any sudden jump in $g(m)$ could be used to signal a unexpected effect around mass m.

Our work is in some sense falsifiable at mid term: If either no particle appears near 115 GeV or QHD proves able to reproduce the spin couplings for double magicity, the coincidence will be invalid in the first case, accidental in the later. A ten-years lapse will do the final judgement, as computing power increases and the LHC does its hunt.

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[14] about or less than 1 percent in magnitude.

[15] indirectly quoted in [4] this year 2003, so it should be still considered state-of-the-art!

[16] thanks to P. Walters for this remark

[17] An intermediate suggestion -to be tested- is to calculate the interaction between the whole neutron and proton balls, with respective masses M_N, M_P, when the interacting particle has $m \approx M_N + M_P$. This is suggested because observation of quenching hints that both subshells must be present for the double magicity phenomena to appear.