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Letter

An interpretation of scalars in SO(32)

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Abstract We propose an interpretation for the adjoint representation of the $SO(32)$ group to classify the scalars of a generic Supersymmetric Standard Model having just three generations of particles, via a flavour group $SU(5)$. We show that this same interpretation arises from a simple postulate of self-consistence of composites for these scalars. The model looks only for colour and electric charge, and it pays the cost of an additional chiral $+4/3$ quark per generation.

1 Introduction

While highly relevant in string theory and supergravity, the $SO(32)$ group is not a good unification group as it doesn't have complex representations [19]. But it still gets an interesting family group when decomposed. In this letter, we first review the decomposition, interpret it as a group symmetry on scalars that could be supersymmetry partners of the Standard Model fermions, and then we present an interesting reconstruction of such scalars as composites. Besides, the interpretation has a uniqueness that limits the number of generations for the SM group.

This reconstruction could have some application when considering open string theory and their branes, or could be used as basis for other GUT-flavour models. Considering this, we include a pair of sections with some separate discussion on other related groups.

2 The flavour group in $SO(32)$

The authors of [18] classify decomposition of groups having explicitly a $SU(3)$ colour subgroup, giving candidate representations as well as the decomposition of the adjoint representation in all the cases. Groups $SO(2n)$ are case 4 of this classification, where they obtain the decomposition $SO(n_1) \otimes SU(n_2) \otimes SU(3) \otimes U_1(1)$ with $2n = n_1 + 6n_2$. Our case of interest is $SO(32)$ with the maximal $SU(n_2)$, this is $n_2 = 5$. The representations intended for fermions are not very useful, as the group is of kind $SO(4k)$, without complex representations. But we are interested on the adjoint as a place for scalars. The stated result gives us

$$
\begin{array}{llll}
496 = & & & \\
(\mathbf{1}, \mathbf{24}, \mathbf{1^c}) & +[\mathbf{1}, \mathbf{15}, \bar{\mathbf{3}}^{\mathbf{c}}] & +[\mathbf{1}, \overline{\mathbf{15}}, \mathbf{3^c}] & + \\
1, 24, 8^c & +[1, 10, \bar{6}^c] & +[1, \overline{10}, 6^c] & + \\
(1, 1, 8^c) & & & + \\
 & (2, 5, 3^c) & +(2, \bar{5}, \bar{3}^c) & + \\
 & (1, 1, 1^c) & +[1, 1, 1^c] &
\end{array}
\qquad (1)
$$

And our components of interest are the three first ones, that we have stressed with boldface. The explicit $U_1(1)$ group provides an hypercharge that counts the number of coloured representations and is zero for colour singlets, so we can assign respectively $Y_1 = 0, +1, -1$ to the above 1^c, $\bar{3}^c$ and $\bar{3}$.

To get a second hypercharge, we can consider $SU(5)$ as the flavour group and decompose it [15,28,29] down to multiplets in $SU(3) \times SU(2) \times U_2(1)$

$$15 = \quad (1, 3)_{-6} + (3, 2)_{-1} + (6, 1)_4 \qquad (2)$$

$$24 = (1, 1)_0 + (1, 3)_0 + (3, 2)_5 + (\bar{3}, 2)_{-5} + (8, 1)_0 \qquad (3)$$

Now from the two hypercharges we can produce a charge

$$Q = \frac{1}{5}\left(\frac{2}{3}Y_1 - Y_2\right) \qquad (4)$$

and check that the resulting decomposition includes content corresponding to the scalars of a minimal, three generations,

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supersymmetric standard model.

	(f_3, f_2)	Y_1	Y_2	Q
$(1, 15, \bar{3})$	$(3, 2)$	1	-1	$+1/3$
	$(6, 1)$	1	4	$-2/3$
$(1, \overline{15}, 3)$	$(3, 2)$	-1	$+1$	$-1/3$
	$(6, 1)$	-1	-4	$+2/3$
$(1, 24, 1)$	$(1, 1)$	0	0	0
	$(1, 3)$	0	0	0
	$(8, 1)$	0	0	0
	$(3, 2)$	0	5	-1
	$(\bar{3}, 2)$	0	-5	$+1$

(5)

Plus an extra content

		Y_1	Y_2	Q
$(1, 15, \bar{3})$	$(1, 3)$	1	-6	$+4/3$

(6)

We can arrive to the same result by chaining some branchings. A straightforward way is $SO(32) \supset SU(16) \times U(1)$,

$$496 = 1_0 + 120_4 + \overline{120}_{-4} + 255_0 \tag{7}$$

and then $SU(16) \supset SU(15) \times U(1)$ and $SU(15) \supset SU(5) \times SU(3)$, to finish applying (2), (3). In this way the quarks come from the initial 120s, while the leptons are from the 255. Or respectively in $SU(15)$, from the 105s and the 224.

3 $SO(32)$ from postulates

Once we know that our aim is to get not the fermions but just the scalar partners of a Susy Standard Model, we can wonder if there is some set of postulates that isolates directly the flavour group, or at least the number of generations it has. It turns out, there is an amusing set of requirements that force this result.

The clue is the "recursive" property of colour: we can get the **3** colour triplet out of $\bar{\mathbf{3}} \times \bar{\mathbf{3}} = \mathbf{3} + \mathbf{6}$. And also we can get singlets, from $\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{1} + \mathbf{8}$.

And adding to this hint, we notice that one quark with an antiquark allows to build particles of electrical charges +1, 0, and -1, but not only that: also we can build a charge +2/3 with two antiquarks of down type, and a charge $-1/3$ with one antiquark down plus other antiquark down. This was in fact the spirit of the above decomposition of $SU(5)$ flavour, but it is even more interesting when starting from particles and going later to groups.

3.1 Turtles and elephants

We consider scalars as composites either of pairs of quarks, as a colour triplet, or of pairs quark anti-quark, as a singlet. Furthermore, we divide the quarks in two classes: *turtles* and

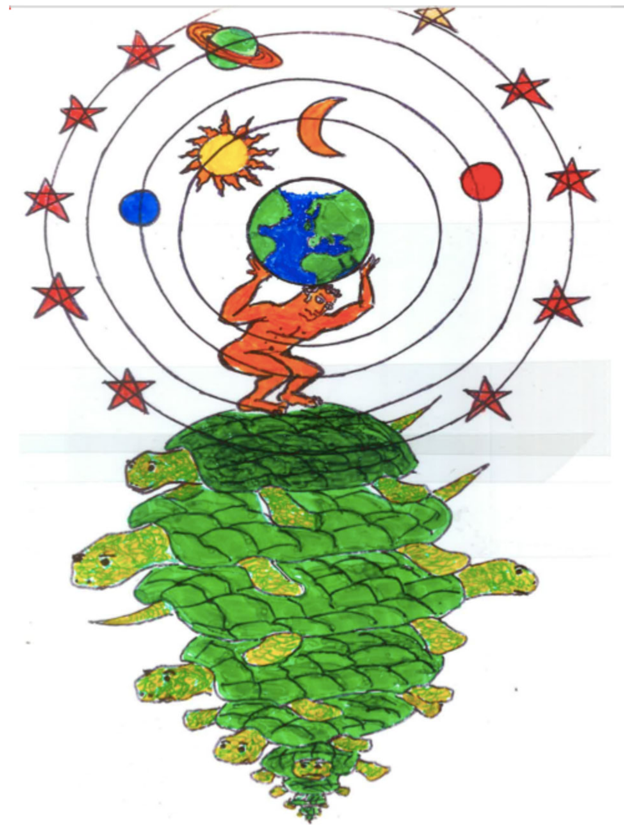

Fig. 1 Illustration of the concept *Turtles all way down*, with an spectator but massive *giant* (Credit of the drawing: De Rújula)

elephants,[1] and add a rule: only *turtles* can combine into composites.

We assume there are N up-type quarks, of these k_u *turtles*, and N down-type quarks, of which k_d *turtles*.

We ask for what values of N, k_u, k_d the number of scalars of each type is exactly $2N$, as required in supersymmetry models. This gives two equations for squarks up and down:

$$2N = k_u k_d \tag{8}$$
$$2N = k_d(k_d + 1)/2 \tag{9}$$

So $N \geq 3$ (actually, N must be half of an hexagonal number) and $k_d = 2k_u - 1$. If we add other two conditions, for sleptons charged and neutral

$$2N = k_u k_d \tag{10}$$
$$4N = k_u^2 + k_d^2 - 1 \tag{11}$$

then the solution is unique, $N = 3$, $k_u = 2$, $k_d = 3$. There are five *turtles* and one *elephant*, that we can name as the top quark.

However, note that if we consider all the combinations of *turtles* we find that we get three extra "squarks" of charge $+4/3$, and their opposites.

1 Or *giants* see Fig. 1.

3.2 Colourless and coloured flavour groups

The extra "squarks" look as a penalisation but group theoretically they are the ones that allow to complete the flavour supermultiplet into a 15 of $SU(5)$

At this level and without colour, we could consider that the flavour is organized in the 54 of $SO(10)$, and then break it down to $SU(5) \times U(1)$

$$54 = 15_4 + \bar{15}_{-4} + 24_0$$

where again the hypercharge from this U(1) can be combined with the one on (2), (3) to reproduce the electric charge.

If we want to incorporate colour and unify colour-flavour, our minimal candidate is $SU(15)$. From here we can go up to $SO(30)$ and then to $SO(32)$ adding singlets, or substituting colour $SU(3)$ by $U(3)$.

4 A case for one generation

The above argument assumed the SM and particularly that the turtles were allowed to bind only if they had colour. But the final particle content is very reminiscent of the Georgi-Glashow model.

Consider the usual formulation of the model

$$5 = (1, 2)_{-3} + (3, 1)_2 \quad (12)$$

$$10 = (1, 1)_{-6} + (\bar{3}, 1)_4 + (3, 2)_{-1} \quad (13)$$

and use $Q = T_3 - Y/6$ to study the electric charge of the colour singlets and colour triplets in each representation of $SU(5)$

Repr	Singlets	Triplets	Antitriplets	Other
1	0			
5	0, +1	−1/3		
24	**0**, +**1**, **0**, −**1**	−4/3, −**1/3**	+4/3,**+1/3**	$(8)_{q=0}$
10	**+1**	**+2/3**, −**1/3**	**−2/3**	
$\bar{10}$	−1	**+2/3**	**−2/3**, **+1/3**	
15	+2, +1, 0	+2/3, −1/3		$(6)_{-2/3}$
$\bar{15}$	−2, −1, 0		−2/3, +1/3	$(\bar{6})_{+2/3}$

It can be argued that the union of $10 + \bar{10} + 24$ is the one generation version of our previous construction, albeit with only a left neutrino. See how the X boson of the GUT model is here just the q = +4/3 scalar, and how we have the condition of two states for each fermion. In this case our turtles, binding, are all the members of 5 and the elephants, not-binding, are the members of 10. They bypass the previous uniqueness proof because we are allowing leptons to join the game.

It could be interesting to consider variants of this game, such as flipped $SU(5)$ [12] with some rules for the $U(1)_X$ charges of the 24 and 15, or even an anomalous "deflipped $SU(5)$" with the up quark in the fundamental but the standard hypercharge assignments, so that the $+4/3$ triplet would appear in the decuplet. The authors in [9] use a different charge assignment to produce the +2/3 charge in the 24, so that it is composed of gluons, a photon, and a whole set of $SU(3) \times U(1)$ charges, and then a broken susy $SU(5)$ produces the standard model particles as Goldstone fermions.

The $SU(5)$ model was frequently used as foundational model in the peak of composite theories in the early eighties. In most cases the components in the 5 were new particles, but some models considered to keep right fermion as elementary and only left as composite, and even there was some proposal [14] were the 5 had a shared mix of preons and known fermions. The author in [4] considers a fundamental "quint" and a composite 10, albeit from the product of three "quints". Early literature also includes ideas where only the leptons are composite, as well as proposals where only the first family is elementary.

5 The role of the top

The argument in Sect. 3 tells us that there is one quark that does not act as a preon for the susy scalars. It does not tell us which one. We need to identify the elephant. And really we have not a concrete argument.

We favour the top quark due to the horizontal, flavour-like, symmetries found in the first section: $SU(3)_{f_3} \times SU(2)_{f_2}$. When considering the values of quark Yukawa couplings, and thus quark masses, it seems more fitting that the f_2 symmetry relates u and c, with the t quark being the excluded one. This scenario is typical when breaking textures.

We were in fact inspired by the empirical observation that toponium doesn't exist, but this is perfectly justified by the mass of the top being closer to the Fermi scale than to the QCD scale. So it disintegrates faster than it binds. However, we have not discussed the binding mechanism here, and the one from QCD doesn't apply.

The heavy mass of the top quark can be used as an argument in compositions where the quarks act as charges at the ends of a relativistic open string, as they need to be massless, or at least as light as possible. And ultimately it can be expected that masses in the standard model -with right neutrinos but not other particles- are protected by two dual symmetries: one that fixes all the degrees of freedom as Dirac massless except the top quark, and another one that fixes all the degrees of freedom as Majorana massless except the neutrinos.[2]

[2] Such protection splits degrees of freedom as 84+12 in both cases, and thus we should look for some group representations in the 84, 42 or 21. Note M2-brane and M5-brane carry a $SU(9)$ symmetry.

6 Discussion on masses

The main point of this section is the existence of a mass formula that produces pairs of equal mass particles, as required by unbroken susy.

Excepting the top quark, preon models from the eighties were known to produce realistic masses. One model from that age is [24], that became popular later due to the accuracy in the lepton sector. It assumes that all the mass comes from a single abelian charge and that all the preons are presented in the same state, so the energy of a pair, having the same spatial wavefunction, is simply the energy of a single element with the sum of the charges

$$E(q_a, q_b) = E(q_a + q_b) = (q_a + q_b)^2 K_\Omega \tag{14}$$

Intuitively one could imagine classical charge on two spherical surfaces of radius Λ and common center, and sum both self-energies plus the interaction energy and see how it does not depend of the radius Λ.

Furthermore, the model [24] makes two provisions for composites of three pairs (a, b^i) to produce realistic masses:

- That the charge of the three b^i particles add to zero

$$z_1 + z_2 + z_3 = 0 \tag{15}$$

- That the self energy of the common preon a averages the self energy of the other three

$$z_0^2 = \frac{z_1^2 + z_2^2 + z_3^2}{3} \tag{16}$$

The first condition is easily met extracting an abelian charge from any direction of a $SU(3)$ triplet, and we have some. The second is a sort of trace condition but it is imposed ad-hoc. Remember also that $z_i^2 = (z_j + z_k)^2$. So at least some scalars keep having the same self-energy than a third preon in the set.

Lets parametrize with an angle α all the possible ways to produce an abelian charge from the 3 representation of SU(3), in T_3, T_8 basis

$$z_1 = \frac{1}{2}\cos\alpha + \frac{1}{2\sqrt{3}}\sin\alpha \tag{17}$$

$$z_2 = 0 - \frac{1}{\sqrt{3}}\sin\alpha \tag{18}$$

$$z_3 = -\frac{1}{2}\cos\alpha + \frac{1}{2\sqrt{3}}\sin\alpha \tag{19}$$

$$z_0 = \pm 1/\sqrt{6} \tag{20}$$

This is T_3 for $\alpha = 0$ and T_8 for $\alpha = \pi/2$.

The solutions have an obvious periodicity $2\pi/3$ and symmetries at $\pi/6$ and $\pi/2$.

As it is well known, if we use a scale factor $k = m_e + m_\mu + m_\tau$ then for $\alpha = 0.745821$ the mass triplet $[k(z_0 + z_i)^2]$ recovers exactly the values m_e, m_μ, m_τ. This is sometimes interpreted as a prediction for m_τ [24], as we can use m_e and m_μ to recover α, k and then calculate the extant mass, well within one sigma of the current measurement error. Note also that $kz_0^2 = 313.85$ MeV, a familiar quantity from QCD [27].

More interestingly, we can ask for the mass values of the octet and sextet. Note that they do not depend on z_0; our choosing of sign is translated to the whole $[z_1, z_2, z_3]$ tuple, but this is in turn just a exchange of preons with antipreons.

6.1 Paired scalars in the same representation

Our goal is to check if we can realistically recover a mass spectrum similar to supersymmetry scalars. This means that we should find pairs of scalars in the same representation having equal masses. We have to cases.

For $\alpha = \pi/2$ or $\pi/6$ we recover two equal masses and a different one in the triplet of (3, 2)

$$\frac{1}{4} \pm \frac{\sqrt{2}}{6}, \frac{1}{2} \mp \frac{\sqrt{2}}{3}, \frac{1}{4} \pm \frac{\sqrt{2}}{6}$$

The octet is never a problem as we have the "antiparticles" in the same set. The sextet fails for one pair. The triplets of (3, 2) are also a source of trouble, as the pair should be expected to happen across the $SU(2)$; the only possibility is to assign the same charge to "preons" c and u.

A better result happens for $\alpha = 0$. Here a change of sign just exchanges charges z_1 and z_3, so we get exact pairs of masses in all cases. More important, also the sextet is grouped into pairs.

So we can conclude that it is possible, with this mass formula, to obtain an spectrum that resembles the scalars of a mildly broken supersymmetry.

If we take seriously that the mass of the fermions is just the "preonic" self energy, then the discrepancy in the $\alpha = 0$ case contains a sort of isospin exchange: we have assigned

$$u = c = 313.85,$$
$$s = 0, d = b = 470.8$$

and we have got

$$(u, c, t) = (0, 470.8, 1883),$$
$$(d, s, b) = (15.8, 313.85, 1553.4)$$

6.2 Solutions with paired scalars in different representation. Missing symmetry

Lets go a bit beyond this the scope of this work, to look also for rotations that create some pairs in different representations. The motivation here is to explore for some extra symmetry that includes quarks of different charges. This is

Table 1 "slepton" masses for $kz_0^2 = 313.85$ MeV. There are always two massless sneutrinos in the octet (8, 1) and other two in (1, 3). The other two combinations of (1, 3) and (1, 1) can be chosen to have zero mass too

		0	$\pi/2$	$\pi/4$	α_{SM}	$\alpha_{.68583}$
(3,2)	$\bar{u}d$	1553.4	914.63	1756.96	1776.9	1801.22
	$\bar{u}s$	313.85	53.848	0	0.5110	3.4179
	$\bar{u}b$	15.853	914.63	126.144	105.65	78.463
$(\bar{3}, 2)$		(as above)				
(8,1)	$\bar{s}d, \bar{d}s$	470.775	1412.32	1756.96	1717.2	1647.7
	$\bar{d}b, \bar{b}d$	470.775	1412.32	126.144	91.47	49.128
	$\bar{b}s, \bar{s}b$	1883.1	0	941.55	1016.0	1127.8
	$(\bar{s}s, \bar{d}d, \bar{b}b)$	0				
	$\bar{u}u, \bar{c}c$	0				
	$(\bar{u}c, \bar{c}u)$	0 if $u = c$, 1255.4 if $u \neq c$				

Table 2 "squark" masses for $kz_0^2 = 313.85$ MeV. Note that a global change of sign in s, b, c exchanges the (3, 2) of sleptons and squarks

		0	$\pi/2$	$\pi/4$	α_{SM}	$\alpha_{.68583}$
(3,2)	ud	15.853	26.93	42.048	45.18	49.128
	us	313.85	1829.25	1255.4	1205.3	1127.8
	ub	1553.4	26.93	585.65	632.66	706.162
(6,1)	dd	1883.1	627.7	2342.61	2388.9	2445.29
	ds	470.775	156.93	42.048	55.31	78.463
	ss	0	2510.8	1255.4	1156.1	1007.05
	bd	0	627.7	313.85	289.03	251.76
	bs	470.775	156.93	585.65	597.21	611.32
	bb	1883.1	627.7	168.192	221.23	313.85

phenomenologically motivated by two approximate observations:

- that the top mass still can fit in a Koide SU(3) tuple but only respect to high masses, for instance (m_c, m_b, m_t).
- that (m_s, m_c, m_b) also look as a valid tuple.

Both observations need to get values from different representations. Looking for such solutions is beyond the scope of this work, but we can cover here the cases where a mass appears in two representations, as it is really a completion of our inspection.

We find two cases. First, for $\alpha = \pi/4 = 0.7854...$ we obtain for the triplet of the sleptonic (3, 2):

$$\frac{1}{4}(2+\sqrt{3}), 0, \frac{1}{4}(2-\sqrt{3}) \tag{21}$$

and with change of sign:

$$\frac{1}{12}(2-\sqrt{3}), \frac{2}{3}, \frac{1}{12}(2+\sqrt{3}) \tag{22}$$

The second case happens when $z_0 = \pm 2z_i$ for some index i. All the solutions are similar obtained as reflections and translations of $\sin^{-1}\frac{1}{2\sqrt{2}}$; we show in the tables $\alpha = 0.6858...$ for continuity.

The proportion (21) was first found by [21] who assigned them to (m_s, m_u, m_d), back in 1978.

It is interesting that if we scale (21, 22) to get equal masses in both tuples, we observe a relation $\sum m_q = 3\sum m_l$, and that works empirically for (m_s, m_c, m_b) with respect to (m_e, m_μ, m_τ). Also a comparison of Table 2 respect to the quantities of Table 1 seems to hint a missed factor of three in other cases: 627.7 → 1883.1, 156.93 → 470.7, 313.85 → 841.55. With the factor three the tuple we have called (ud, us, ub) gets more realistic masses and we know from elsewhere that it can be further rotated to produce more exact values, but we will not pursue this here[3]

Our conclusion in this section is that the breaking down to a single $SU(3)$ flavour, and particularly the neglect of weak isospin, misses some extra symmetry that would allow more comprehensive mass formulae.

7 Discussion on related groups

7.1 On $SU(15)$

For the group decomposition, similar results could be obtained with only $SO(30)$ or $SU(15)$ as a coloured flavour group, or $SO(10)$ or $SU(5)$ as colourless flavour groups, or even with $Usp(32)$.

$SU(15)$ was considered as a GUT group by [1] and [16]. The first reference notes that it is a subgroup of $SO(32)$ Both references embed a full generation

$$(l_L, l_L^c, \nu_L, u_{rgb,L}, u^c_{rgb,L}, d_{rgb,L}, d^c_{rgb,L})$$

inside the fundamental of $SU(15)$. On the other hand, our approach embeds the (2, 1) + (1, 3) *turtles* of our $SU(5)$ flavour:

$$(u_{rgb}, c_{rgb}, d_{rgb}, s_{rgb}, b_{rgb})$$

and we use, as noted above, the 105, $\bar{105}$ and 224 representations.

Recently [10,11] have considered $SU(15)$ in the context of the standard model extended with bifermions, so they

[3] It is just an empirical observation, that solving

$$m_3 = \left(\left(\sqrt{m_1}+\sqrt{m_2}\right)\left(2-\sqrt{3+6\frac{\sqrt{m_1 m_2}}{\left(\sqrt{m_1}+\sqrt{m_2}\right)^2}}\right)\right)^2$$

with input (172.4, 4.183) gets 1.3495, and then input (4.183,1.3495) gets 0.092.

naturally use these representations. They consider the particles to be elementary, so "biquarks" instead of "diquarks" or mesons, but this distinction blurs away when we consider an interpretation as open string terminated in quark labels. More importantly, they still keep having leptons in the fundamental representation, so it is possible to get a lepton number in the 15×15 and $15 \times \bar{1}5$ products.

The difference with our model is due to option for the breaking path $SU(15) \supset SU(12) \times SU(3)_l \times U(1) \supset SU(6)_L \times SU(6)_R \times SU(3)_l \times U(1) \times U(1)$, that allows to put a whole generation of the SM without right neutrinos in the decomposition of the 15, at the cost of some delicate surgery [1,16]. The first extracted $SU(3)_l$ group has the goal of joining all the leptons of each generation in a single multiplet; if we want an extra ν_L^c neutrino it must be expanded to $SU(4)_l$ and then the whole group to $SU(16)$

7.2 On $SU(8)$

This section and the next one are explorative work, the main theme being if representations of other groups from supergravity and string theory can benefit of a similar interpretation as scalars of some supersymmetric standard model.

$SU(8)$ appears directly because an alternate chain down from $SO(32)$ is to take the detour $SU(16) \supset SO(16) \supset SU(8) \times U(1)$

$$\begin{aligned} 496 &= 1 + 120_4 + 120_4 + 120_0 + 135_0 \\ &= 1 + 3(1_0 + 28_2 + \bar{2}8_{-2} + 63_0) \\ &\quad +36_2 + \bar{3}6_{-2} + 63_0 \end{aligned} \tag{23}$$

And then we can go for the group theory of $SU(8) \supset SU(5) \otimes SU(3) \otimes U(1)$ but with a lot more of hypercharge assignments (usually uglier, but worth a glance).

Family GUT unification with $SU(8)$ was examined with some detail in 1980, see for instance the references in the recent revisit of [3]. Typically three families of standard model fermions were expected to be in the summed complex representation $\bar{8} + \bar{2}8 + 56$ and some criteria was used to select the hypercharge assignments.Most models preferred to interpret for flavour the first $SU(3)$ in $SU(8) \supset SU(5) \otimes SU(3) \otimes U(1)$ instead of leaving it for colour as [18]. Both approaches differ only in the algebra of $U(1)$ charges for the multiplets. The fundamental decomposes as a colour triplet, a $SU(2)$ horizontal doublet, and a $SU(3)$ horizontal triplet.

$$8 = (1, 1, 3)_{0,-5} + (1, 2, 1)_{-3,3} + (3, 1, 1)_{2,3}$$

Note it went first to

$$8 = (1, 3)_{-5} + (5, 1)_3$$

and while in the first approach $SU(5)$ is flavour-colour, in the second it is just two horizontal symmetries and the colour triplet is explicit. So we prefer this later way because so all the irreducible representations of $SU(8)$ have an interesting interpretable descent. The decomposition of the 28 has a quark content, triplets, that looks very much as our division in five turtles and one elephant,

$$28 = (1, \bar{3})_{-10} + (5, 3)_{-2} + (10, 1)_6$$

but it is different to the $SO(32)$ case. To illustrate a particular assignment, if we think of the fundamental as "half-charged preons" of charges $\pm 1/2$, $1/6$, then in the 28:

- $(1, \bar{3})$ is one anticoloured particle of charge $+1/3$
- $(5, 3)$ are coloured particles, three of charge $-1/3$, two of charge $+2/3$
- $(10, 1)$ contains six particles of charge 0, three of charge -1 in an horizontal "antitriplet"…and one of charge $+1$

So this content doesn't allow for our "recursive" interpretation of the interplay between the 32 and the 496 of $SO(32)$

We can play also with content from extra representations. The 36 somehow complements the 28, and the 63 can provide a full uncoloured (24, 1) to break into different charges. Besides, in this path, the fundamental of $SO(32)$ appears in $SU(8)$ as

$$32 = (8_{1,2} + \bar{8}_{-1,2}) + (8_{1,-2} + \bar{8}_{-1,-2}) \tag{24}$$

and so it provides extra $U(1)$ charges and extra particles; one needs a good motivation to justify a particular pick. We can explore one hundred weightings to extract the electric charge Q of each representation, most of the combinations offering extra quark and lepton content, including some $+4/3$ quarks.

We could also use the process via via $SU(5) \supset SU(2) \otimes SU(3)$ to assign weak and colour multiplets as usual. On our point of view, both $SU(2)$ and $SU(3)$ here are horizontal groups.

One can observe that (24) meets the condition asked in [18] of having only singlets and triplets of colour, and so wonder what reasons, besides simplicity, motivate the exclusion from the listing.

We could also consider first a regular descent, via $SO(16)$ to $SU(8) \times SU(8)$

$$\begin{aligned} 120 &= (8, 8)_0 + (28, 1)_2 + (1, 28)_{-2} \\ 255 &= (1, 1)_0 + (8, \bar{8})_2 + (\bar{8}, 8)_{-2} + (63, 1)_0 + (1, 63)_0 \end{aligned}$$

7.3 On $E_8 \times E_8$

Exotic approaches to flavour are not unknown in supergravity, a good example being the diagonal $SU(3)$ from Gell-Mann, that also ignores electroweak charge [26]. And as $SO(32)$ is relevant to 10D sugra, and all the 10D supersymmetric theories are related via string dualities, it is interesting to speculate if other corner of this space, the $E_8 \otimes E_8$ group, can present a similar mix.

We can examine this possibility starting from the conclusions of the above sections, albeit at the moment the discussion will be very light, and inconclusive, if not disappointing.

E_8 is not considered in [18] because the authors apply a "colour restriction" in their selection of groups, asking for decomposition of the fundamental representation having only singlets and triplets of $SU(3)$. It is more particularly reviewed by [2], who enumerates the problems to use it as a group GUT and also considers decomposition with explicit family group $SU(3)_F$. A separate approach with explicit colour group $SU(3)_c$ and then mixed electroweak-flavour $SU(6) \times U(1)$ was done in [7] via an initial breaking into $SU(9)$. Generically, E_8 has an industry of its own for pure algebraic approaches, linked to Clifford algebras, and full of interesting observations, but reviewing it is out of the scope of this letter.

Both $SO(32)$ and $E_8 \otimes E_8$ have a subgroup $SO(16) \otimes SO(16)$. The branching of $SO(32)$ to this subgroup is

$$496 = (120, 1) \oplus (1, 120) \oplus (16, 16)$$

very similar to the branching we have used in (7)

Isolately, each E_8 branches to $SO(16)$ as

$$248 = (120) \oplus (128')$$

What we suspect is that quark and lepton parts have different roles, the quark part coming from 120; one of the 120s will provide the quark-like charges, the other will provide the antiquark ones. The lepton part can be extracted from the 28 of $SU(8)$ but it could also come from the 63, and then we should investigate the $(128')$ irrep.

Remember that in the initial sections the critical part has been to obtain a 15 representation of $SU(5)$ associated to a triplet 3 of $SU(3)$, as well as a 24 associated to the singlet. And here is the problem: any further factorisation of $SO(16)$ fails to get representations as big as the 15. We are down to fives and tens too soon. Amusingly, we could also consider a directly branching $E8 \supset SU(5) \otimes SU(5)$; this is exploited in model building, for instance [3,8], but with different assignments to colour and flavour. If we use this kind of decomposition and we accept the irreps 5 and 10 instead of the 15, it amounts to exchange some of the $\pm 4/3$ and $\pm 2/3$ charges by an excess of $\pm 1/3$ charges.

8 Discussion

The postulate *It is turtles all the way down*[4] applied solely to squarks already fixes the number of generations to be greater or equal than three. Adding a reasonable condition on the building of neutral sleptons, it fixes uniquely $N = 3$ and then also the separation between five light quarks and one heavy one that does not participate in the composites. Of course this uniqueness is not seen when going directly from the $SO(32)$ group down to flavour times colour, but even in this case there is a separation between five "turtles" in the fundamental of $SO(32)$ and a non-participant "elephant".

While eventually all the extant multiplets of the decomposition should be explained, the $(1, 3)$ squarks, of charge $\pm 4/3$, are specially puzzling. They can not be organised as three generations of partners of four-component Dirac quarks. Still, they have a role in the flavour multiplet, and they could exhibit their singularity if chirality is introduced back in the game. We have not considered other flipped descents that could result in different extra *squarks*

The symmetry between quarks and diquarks or its hadronic equivalent is known to be one of the historical origins of supersymmetry [17,25] and it is used in hadronic phenomenology. But a concrete hadronic construction of our scalars as real diquarks produces the ones of odd parity, that are excluded of phenomenological discussions as they do not survive the 'single mode approximation' [23]. Thus the composite "squarks" and "sleptons" bound here should be not the ones, diquarks and mesons, found at QCD scale, but it is intriguing that they are similar in number and mass.

Approximate supersymmetry between quarks and diquarks, to be used for dynamical supersymmetry between barions and mesons, is a well known technique, see the review [5]. One of the authors of this review extended this technique to preons [13], so that dipreons are susy partners of preons, but the model does not produces a dynamical susy with new scalars as partners of leptons and quarks.

We can justify the uplift from $SU(15)$ to $SO(32)$ by asking particle colour to be in a slightly greater group, such as $U(3)$. This could be a hint of the difference between the binding mechanism needed here, that should happen at high energy scale, and the usual binding of mesons and diquarks. Observe that the usual binding shares some properties: the top quark, our *elephant*, does not bind into mesons -because it disintegrates before-, and the masses of mesons and diquarks are in the same range of energies that the SM fermions, as expected of a slightly broken supersymmetry.

While the composites point to HC, TC or ETC models, one must recognise that the motivation for $SO(32)$ is not only to produce one hypercharge and the adequate multiplets in the decomposition, but also because of its role in string theory. The postulates of composition need a pairing that looks similar to labels in terminated open strings. The composition process from the point of view of a terminated "QCD string" bears some similarity to the techniques of [6] using "planar orientifolds".

[4] I first heard this idiom in a talk from Alvaro de Rujula in 1986.

If we get the scalars from $SO(32)$ a natural question is where the superpartners -the actual fermions- are. They could be obtained by applying the susy transformation. In string-inspired GUTs, they should be in usual non-susy models for three generations, and the issue would be to reassure the compatibility with the $SO(32)$ group. In preon constructions, our focus on scalars, and thus in pairs of fermions, makes the results to differ from most previous approaches [13,14,20]. To recover a fermion, one must consider an extra object, particle or string, providing again an 1/2 spin.

9 Conclusions

In conclusion, lets review what we have got. We offer a novel interpretation of the $SO(32)$ group within the context of supersymmetric models, emphasizing its potential as a flavour group for scalars. The decomposition and hypercharge assignment that allows to recover three generations has not been presented in the literature explicitly. This is for the obvious reason that it recovers scalars, not fermions. But on the other hand, to look for scalars avoids to address the problem of the lack of chiral fermions.

Besides, we offer a composite explanation for scalars of the SSM, that fixes the number of generations and limits the possible groups that can be used to generate flavour with a separate colour factor. In the list of possible groups, $SO(32)$ stands up.

Our postulate is, certainly, exotic: it suggests that while SSM fermions could be elementary, the SSM scalars are composites, with their preons being a subset of the observed SM fermions. Far fetched as this postulate looks, it reproduces the $SO(32)$ decomposition and fixes the number of possible generations. Also, it justifies the non observation of superpartners; supersymmetry could be hiding in plain sight, not broken but distorted.

We have recalled a classical mass formula that when applied to our piece of $SU(3)$ flavour symmetry can grant pairs of scalars having the same mass.

The decomposition seems to imply that each generation also includes two extra "scalar quarks" of charge $\pm 4/3$. It is unclear if such scalars could have an associated fermion, as it should be of Weyl type, not Dirac. On other hand, our approach could be compared to the "scalar democracy" of [22] that goes further and proposes the existence of a scalar bound state for every pair of fundamental fermions, either leptons or quarks.

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